

Exercise 45

(a) Estimate the value of

$$\lim_{x \rightarrow -\infty} (\sqrt{x^2 + x + 1} + x)$$

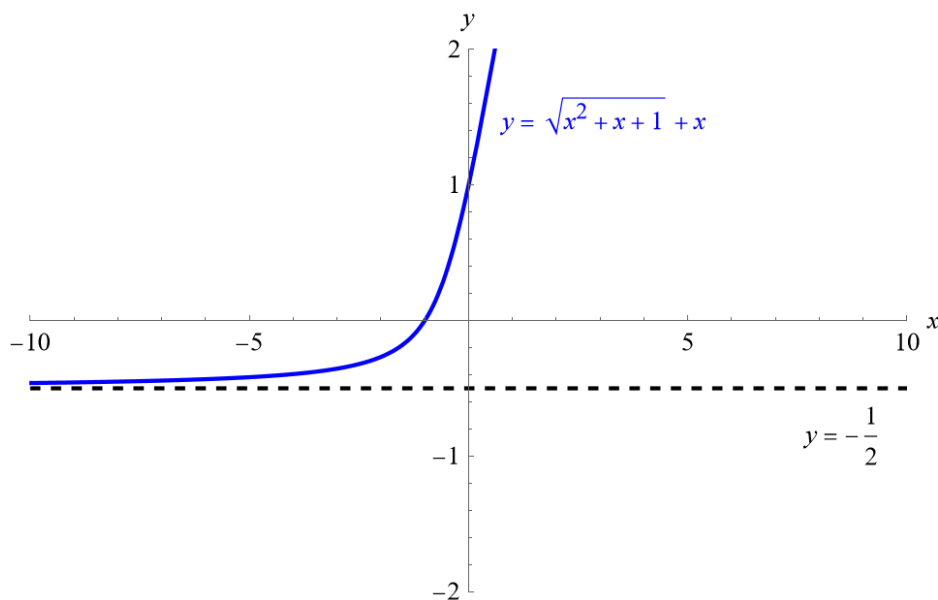
by graphing the function $f(x) = \sqrt{x^2 + x + 1} + x$.

(b) Use a table of values of $f(x)$ to guess the value of the limit.

(c) Prove that your guess is correct.

Solution**Part (a)**

Graph the function versus x .



Based on the graph, the limit of the function as $x \rightarrow -\infty$ is -0.5 .

$$\lim_{x \rightarrow -\infty} (\sqrt{x^2 + x + 1} + x) = -\frac{1}{2}$$

Part (b)

Make a table with large negative values of x to see what happens as $x \rightarrow -\infty$.

x	$f(x)$
-1	0
-10	-0.4606
-100	-0.4962
-1 000	-0.4996

Part (c)

$$\lim_{x \rightarrow -\infty} \left(\sqrt{x^2 + x + 1} + x \right)$$

Start by making the substitution, $u = -x$. Then as $x \rightarrow -\infty$, $u \rightarrow \infty$.

$$\begin{aligned} \lim_{x \rightarrow -\infty} \left(\sqrt{x^2 + x + 1} + x \right) &= \lim_{u \rightarrow \infty} \left[\sqrt{(-u)^2 + (-u) + 1} + (-u) \right] \\ &= \lim_{u \rightarrow \infty} \left(\sqrt{u^2 - u + 1} - u \right) \\ &= \lim_{u \rightarrow \infty} \left(\sqrt{u^2 - u + 1} - u \right) \times \frac{\sqrt{u^2 - u + 1} + u}{\sqrt{u^2 - u + 1} + u} \\ &= \lim_{u \rightarrow \infty} \frac{(u^2 - u + 1) - u^2}{\sqrt{u^2 - u + 1} + u} \\ &= \lim_{u \rightarrow \infty} \frac{-u + 1}{\sqrt{u^2 \left(1 - \frac{1}{u} + \frac{1}{u^2} \right)} + u} \\ &= \lim_{u \rightarrow \infty} \frac{-u + 1}{u \sqrt{1 - \frac{1}{u} + \frac{1}{u^2}} + u} \\ &= \lim_{u \rightarrow \infty} \frac{-u + 1}{u \left(\sqrt{1 - \frac{1}{u} + \frac{1}{u^2}} + 1 \right)} \\ &= \lim_{u \rightarrow \infty} \frac{-1 + \frac{1}{u}}{\sqrt{1 - \frac{1}{u} + \frac{1}{u^2}} + 1} \\ &= \frac{-1 + 0}{\sqrt{1 - 0 + 0} + 1} \\ &= -\frac{1}{2} \end{aligned}$$